Maximal frequency, amplitude, and elbow joint stiffness in cyclic movements

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1. Introduction

Most of research projects on cyclical motor activity focus on locomotion. They provide ambivalent data on the considered theme. Saito et al. [1] present the results of their research on the relationship between step length and frequency in sprint, which was not a decreasing one. Laurent and Pailhous [2] observed that step length and frequency both correlate with walking speed but are relatively independent of each other. Martin et al. [3] observed an inverse relationship between crank length (amplitude) and pedaling frequency. Peper and Beek [4] noticed a slight amplitude decrease with frequency increase of forearm movements. The question may thus be asked whether cyclical movements performed by the human motor system are characterized by an unequivocal relation between movement amplitude and frequency. If this is the case, then what is the nature of this relationship, within what amplitude or frequency ranges and in what way can it manifest itself in complex movements?

The objective of this study derives from the questions posed above and consists in researching hypothetical relations between the amplitude of cyclical joint movement and its frequency, as well as the conditions in which such relations exist and, possibly, the mechanisms lying at the basis of such relationships.

2. Methods

The research was conducted on 11 students, aged 21 ± 1.4 years, with average body mass of 74.9 ± 4.6 kg and height of 1.82 ± 0.04 m. The subjects were instructed to execute cyclical flexion-extension elbow movements in horizontal plane with the highest possible frequency $f_M$ within a defined (approximately) joint angle around central value $\alpha_0 \approx 1.8 \text{ rad}$. Each subject executed two series of movements for five values of the joint angle: ~0.1 rad; ~0.4 rad; ~0.8 rad; ~1.0 rad and 1.2 rad. The value directly measured was the relationship between joint angle and time $\alpha_j(t)$. A precise linear potentiometer was used (nonlinearity error $\delta = \pm 0.5 \%$ and smoothness 0.1 %). The measurements were carried out using a 12 bit A/D converter with a sampling frequency of $f_p = 128 \text{ Hz}$. Angular velocity $\omega(t)$ and acceleration $\varepsilon(t)$ (and their maximal amplitudes: $\omega_M$ and $\varepsilon_M$) were obtained by numerical differentiation of $\alpha_j(t)$.

3. Results

The empirical amplitude-frequency relationship represents a shifted hyperbola described as:

$$\alpha_M = \frac{3.048}{f_M} - 0.362, \quad R^2 = 0.97 \quad (1)$$

Equation (1) shows that for a certain movement frequency $f_{\text{MAX}}$ the amplitude value is $\alpha_M = 0$. The $f_{\text{MAX}}$ is thus the limiting frequency setting the upper limit of the frequency band at which cyclical elbow joint movements can be executed. This limiting frequency was $f_{\text{MAX}} = 8.4 \text{ Hz}$ in the investigated group.

Fig. 1 Relationship between angular acceleration $\varepsilon$ and joint angle $\alpha_j$ recorded in cyclical forearm movements at two different amplitudes $\alpha_M$ and frequencies $f_M$.

In accordance with previous observations [5] the nature of the relationships presented in Figure 1 points to a considerable similarity of the studied movements to harmonic movements, which is confirmed by the linear (for low amplitudes and high frequencies) and nearly linear (for high amplitudes) relation between $\alpha(t)$ and $\omega(t)$. 

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4. Discussion

Fig. 1 shows that the functional component of torque generated at the joint is directed opposite to angular displacement $\alpha = \alpha_j - \alpha_0$, and its instantaneous value is proportional to $\alpha$. Consequently, it acts like a restoring torque produced by linear joint stiffness $K_s$ – the relation between $\varepsilon$ and $\alpha$ can be described as follows: $\varepsilon = -c \cdot \alpha$, while $M_s = I \cdot \varepsilon$. Hence: $M_t = I \cdot \varepsilon = -I \cdot c \cdot \alpha = -K_s \cdot \alpha$.

Coefficient $K_s = I \cdot c$ can thus be regarded as a “substitute joint stiffness”, related to functional torque $M_u$ active in the joint. The results obtained in the present work indicate that the value of the functional torque is proportional to the angular deflection $\alpha$ and its maximal value shows a relation with movement frequency. On account of equivalent stiffness $K_s$ related to torque $M_u$, the limb can be regarded as a second-order system with an eigenfrequency $\omega_0$. This frequency depends on the system’s stiffness and limb inertia:

$$\omega_0 = \sqrt{\frac{K_s}{I}}$$  \hspace{1cm} (2)

It can be modified by varying the value of $K_s$ (according to the needs or conditions in which the movement is executed) within a limited range of values. This way of executing a movement, consisting in initiating an oscillating joint movement with a frequency near to the eigenfrequency, presents an advantage of a favorable relation between the cost (applied torque) and the result – movement amplitude [6]. Consequently, relationship (1) can, if we take into account relationship (2), be represented as follows:

$$\alpha_M = \alpha_{\text{MAX}} \cdot \frac{1}{2\pi f_M \sqrt{I}} \cdot \sqrt{\frac{1}{K_s}} \cdot \sqrt{\frac{\varepsilon}{K_s}}$$  \hspace{1cm} (3)

The relationship between the stiffness $K_s$ appearing in cyclical forearm movement and movement amplitude is presented in Figure 2.

![Fig. 2 Relationship between elbow joint stiffness $K_s$ and amplitude $\alpha_M$ of cyclical forearm movement.](image)

The nature of the relationship between movement amplitude $\alpha_M$ and frequency $f_M$ (relationship (1)), in which frequency is raised to the first power, and the relatively low inter-subject scatter of the recorded characteristics suggests that their form results from objective laws underlying movement execution and not simply from the subjects’ individual motor qualities. Moreover, research results point to the existence of a relationship between muscle tension and stiffness [7, 8] which means that this way of actuating cyclical movements is made possible by objective factors.

References