Mathematical modeling of a blood stream in large arteries

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1. Introduction.

A large number of mathematical models of haemodynamics were proposed till now. Kline and Allen considered the relationship of pressure gradient to blood velocity based on a continuum theory of blood [1]. Streeter and Keitzer described pulsatile pressure and flow through distensible vessels [2]. Morgan, Kiely considered wave propagation in a viscous liquid contained in a flexible tube [3]. Kizilova investigated pressure have propagation in liquid-filled tubes of viscoelastic material [4]. However, most of them have one common shortcoming. Usually a model describes one or two characteristic features, but neglects the other ones. The main goal of the proposed model is to combine achievements of deferent models in such a degree, which makes it possible to solve the complicated compound problem.

2. Method

The method of mathematical modeling is used. It leans upon a mechanical model of the object and process and followed by a numerical model.

2.1 Mechanical model

The mechanical model was constructed for the ascending of aorta. The blood is considered at a homogeneous incompressible non-Newtonian (dilatants) liquid. The blood vessel as a circle cylinder with a thick wall consisting of a hyperelastic orthotropic material. The liquid rheology is described by the Power low. At the initional (origin) moment the blood is supposed to be at a rest. A blood motion is initiated by means of a pressure change on the aorta entrance, which is the result of the aortic valve opening during the systole. For this purpose a blood velocity distribution along the entrance cross section is preset as a time function. An equality of the pressure and normal component of velocity is assumed on the blood-vessel boundary as a condition of continuity.

2.2 Mathematical model

The general mathematical setting of the hydroelastic problem consists of the equations, describing a liquid, and equations, describing the deformable vessel material with additional conditions the contact surface. The equations, describing the liquid in the cylinder coordinates with regard to axial symmetry are as follows:

\[
\begin{align*}
\frac{\partial \nu_r}{\partial t} + \nu_r \frac{\partial \nu_r}{\partial r} + \nu_z \frac{\partial \nu_r}{\partial z} &= g - \frac{1}{\rho} \frac{\partial p}{\partial r} + k \left( \frac{\partial}{\partial r} \left( 2A \frac{\partial \nu_r}{\partial r} \right) + \frac{\partial}{\partial z} \left( A \left( \frac{\partial \nu_r}{\partial r} + \frac{\partial \nu_z}{\partial z} \right) \right) \right) + \frac{2A}{r} \left( \frac{\partial \nu_r}{\partial r} - \frac{\nu_r}{r} \right) \frac{\partial \nu_z}{\partial z} \\
\frac{\partial \nu_z}{\partial t} + \nu_r \frac{\partial \nu_z}{\partial r} + \nu_z \frac{\partial \nu_z}{\partial z} &= - \frac{1}{\rho} \frac{\partial p}{\partial z} + k \left( \frac{\partial}{\partial r} \left( A \left( \nu_r \right) + \frac{\partial \nu_z}{\partial z} \right) \right) + \frac{2A}{r} \frac{\partial \nu_z}{\partial z} + A \left( \frac{\partial \nu_r}{\partial r} + \frac{\partial \nu_z}{\partial z} \right) \\
\frac{\partial \nu_r}{\partial r} + \frac{\nu_r}{r} + \frac{\partial \nu_z}{\partial z} &= 0
\end{align*}
\]

where \( A = \left[ \left( \frac{\partial \nu_r}{\partial r} \right)^2 + \left( \frac{\nu_r}{r} \right)^2 + \left( \frac{\partial \nu_z}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \), \( n, k \) - the model parameters, which should be obtained as experimental data, \( \rho \) and \( p \) - the density and pressure of liquid.

The equations, describing a wall material, can be written in the following manner:
The problem is to define (calculate) the velocity vector and pressure in all points of the considering region of liquid and also deformations and strains in each point of the vessel wall.

2.3 Numerical model

The numerical solution of the problem was obtained by means of the computer program “Fluent”, where the network method is used. For the construction of program packet a region of calculation the graphic application “Fluent-Gambit”. The network was constructed in the 3D region and had a concentration near the wall. Besides, the method of deformable mesh, which may be used and effective for the cases of comparatively small displacement of the wall. This method was incorporated into the general computer program.

3. Results

Two main results were obtained. The first is the propagation of deformation and stress in along the wall, which is graphically represented on Fig.1. The second is distribution of velocity along the cross-section, which is presented on Fig.2.

Fig.1 Propagation of deformation in along the wall

Fig.2 Distribution of velocity

4. Discussion

The proposed model describes the propagation of the pulse wave along the ascending section of aorta in normal case. The next step will be a modeling of different pathology cases as in vessel wall, so in input pressure-velocity distribution. For this purpose additional experimental investigations should be done.

References